Propagating of chirped gray solitons in weakly nonlocal media with parabolic law nonlinearity and spatio-temporal dispersion

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In our research, we have examined the existence and stability of chirped periodic and solitary waves in a weakly nonlocal nonlinear medium, which exhibits various types of effects, including inter-modal dispersion, nonlinear dispersion, detuning, spatio-temporal, and parabolic law nonlinearity. By studying the nonlinear Schrödinger equation that describes the field dynamics in this system, a class of nonlinearly chirped periodic waves is derived in the presence of all physical processes. In addition, we have obtained solitary waves of the gray type in the long-wave limit of these nonlinear waveforms. We have found that the frequency chirp associated with these optical waves depends on their intensity and its magnitude can be controlled by manipulating the nonlinear dispersion parameter. Furthermore, we have numerically studied the stability of the gray soliton solution under finite initial perturbations. Our results indicate that the nonlinear waves we have identified represent new types of extremely robust chirped localized structures in weakly nonlocal nonlinear parabolic law media.

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1. Introduction

Envelope solitons are remarkable pulses or wave packets that can maintain their shape while traveling over very long distances. The formation of such nondispersing wave packets arises when the broadening of the pulse due to group-velocity dispersion is exactly balanced by the compression caused by the Kerr nonlinearity [1–5]. The discovery of solitons was first predicted theoretically [6,7] and then demonstrated experimentally [8]. Since then, interest in the study of these localized waves has grown considerably over the past few decades with a view to their potential applications in optical communication systems [9–12].

The nonlinear Schrödinger (NLS) equation governs soliton pulse transmission in optical fibers with durations in the picosecond range. This model is completely integrable using the inverse scattering scheme [13]. Note that the NLS model supports bright and dark solitons, which exist in the normal and anomalous dispersion regimes, respectively [14]. These fundamental solitons are chirp-free pulses because the chirp produced by group velocity dispersion is balanced by the chirp produced by the Kerr nonlinearity [15]. For even shorter pulse durations and higher peak powers, however, the higher-order effects come into play and consideration of their contributions become necessary. The inclusion of such additional effects in the envelope equation leads to the NLS family of equations with higher-order terms, which generate soliton pulses applicable to the subpicosecond or femtosecond regime. The propagation of femtosecond optical solitons in nonlinear fibers is of particular interest due to their advantages in long-distance transmission, faster switching, and high bit rates [16,17].

Instead of chirp-free localized waves, more recently, significant effort is being directed towards examining the formation of envelope solitons with nonlinear chirp in optical fibers and waveguides...
These solitons are highly desirable for their potential to achieve effective pulse compression or amplification [22]. Notably, chirped pulses are useful in the design of optical devices such as fiber-optic amplifiers, optical pulse compressors and solitary wave based communication links [19,23]. In this setting, significant results have been obtained with recent studies analyzing the propagation of non-linearly chirped solitons in optical materials exhibiting different higher-order effects such as quintic nonlinearity and self-steeping processes [18–21]. However, the dynamical behavior of chirped solitons under the combined influence of spatio-temporal dispersion, nonlinear dispersion and weak nonlocality has not been reported yet. Noting that previous studies of non-linear waves have demonstrated that in addition to group-velocity dispersion, the effect of spatio-temporal dispersion on the wave dynamics is significant and should not be neglected [24–27].

Our research focuses on investigating the behavior of periodic and solitary waves in an optical waveguiding medium that demonstrates spatio-temporal dispersion, nonlinear dispersion, parabolic law nonlinearity, and weak nonlocality. This is the first work that explores the existence and propagation characteristics of such waves in such a medium. We will demonstrate that nonlinear dispersion can induce chirping in the nonlinear waves, and the magnitude of frequency chirp is highly dependent on the intensity of the wave.

The main structure of our paper is as follows. In Section 2, we present the NLS equation modeling the wave propagation in a weakly nonlocal medium with spatio-temporal and parabolic law nonlinearity. We also present the general chirped traveling wave solution and its corresponding frequency chirp. In Section 3, we provide chirped periodic and solitary wave solutions of the model and determine the frequency chirp associated with these nonlinear waves. In Section 4, we analyze the stability of our chirped gray solutions against small perturbations. Finally, in Section 5, we summarize our findings.

2. Model and chirped traveling waves

The pulse propagation in a weakly nonlocal nonlinear medium with parabolic law nonlinearity and spatio-temporal dispersion is governed by the higher-order NLS equation [28]:

\[ iq_t + \alpha q_{xx} + \beta q_{xt} + \gamma \|q\|^2 q + \lambda \|q\|^4 q + i \sigma \|q\|^2 q_x + i \epsilon q_{xx} + \mu (\|q\|^2)_x q = 0. \tag{1} \]

In this equation, \( q \) denotes the wave profile, and \( t \) and \( x \) represent the time and distance in sequence. The coefficients \( \alpha, \beta, \gamma, \lambda, \delta, \sigma, \epsilon, \) and \( \mu \) correspond to the group velocity dispersion, spatio-temporal dispersion, cubic nonlinearity, quintic nonlinearity, detuning, nonlinear dispersion, inter-modal dispersion, and weak nonlocality, respectively.

It is worth to point out that \( \mu \) in Eq. (1) can be determined by the medium response function \( R(x) \), and is given by [29,30]:

\[ \mu = \frac{1}{2} \int_{-\infty}^{\infty} x^2 R(x) dx. \]

This nonlocality parameter influences the refractive index change \( \Delta n \), which for the case of weakly nonlocal media with cubic-quintic nonlinearity, has the form of [30]:

\[ \Delta n(l) = \gamma l + \lambda l^2 + \mu \Delta^2 l, \]

with \( l = \|q\|^2 \) being the light intensity. This change in the refractive index involves the nonlocal contribution \( \mu \Delta^2 l \), which gives rise to the last term in the envelope equation (1).

As an important generalization, the higher-order NLS equation (1) incorporates, besides the group-velocity dispersion, cubic-quintic nonlinear terms, and nonlocal effect that constitute the cubic-quintic NLS equation with weak nonlocality [30], the spatio-temporal dispersion (the effect of space-time coupling [31]) and nonlinear dispersion. These additional terms appear as corrections to the slowly varying envelope approximation [32], when considering the propagation of ultrashort light pulses. In particular, the spatio-temporal dispersion term \( q_{xx} \) in Eq. (1) should not be neglected in the (femtosecond) regime of very short pulses, for which the pulse envelope may contain only a few optical cycles [32].

We should point out that chirp-free combined optical soliton solutions of the higher-order NLS model (1) have been obtained under certain parametric conditions in Ref. [28]. In contrast, in the present study, we focus on the existence and properties of periodic and soliton solutions with nonlinear chirp for Eq. (1) in the presence of all physical processes. Interestingly, chirp is very useful in the process of light pulse compression and found potential applications in optical communication systems [33–36].

To obtain the exact chirped wave solutions of the model (1), we consider a solution of the form [37]:

\[ q(x, t) = U(\xi) \exp[i(\theta(\xi) + \omega t)], \tag{2} \]

where the real amplitude \( U(\xi) \) and phase modification \( \theta(\xi) \) are functions of the traveling coordinate \( \xi = x - vt \), with \( v \) is the velocity of the wave. Also, \( \omega \) represents the frequency of the wave oscillation.

By substituting (2) for \( q(x, t) \) into the model (1) and separating the real and imaginary parts, one gets:

\[ (\alpha - \beta \nu) U_{\xi \xi} + (\delta - \omega) U \]

\[ + (\nu - \beta \omega - \epsilon) \nu \theta_x - (\alpha - \beta \nu) U \theta_{\xi}^2 + \gamma U^3 + \lambda U^5 \]

\[ - \sigma U^3 \theta_x + \mu U (U^2)_{\xi \xi} = 0, \tag{3} \]

and

\[ (\alpha - \beta \nu)(U_{\xi \xi} + 2_U \theta_{\xi}) - (\nu - \beta \omega - \epsilon) U_\xi + \sigma U^2 U_\xi = 0, \tag{4} \]

where the subscripts indicate partial derivatives. We can multiply Eq. (4) by \( U(\xi) \) and integrate over \( \xi \) to obtain:

\[ \theta_x = -\frac{\sigma}{4(\alpha - \beta \nu)} U^2 + \frac{A}{2(\alpha - \beta \nu) U^2} + \frac{v - \beta \omega - \epsilon}{2(\alpha - \beta \nu)}, \tag{5} \]

where \( A \) is an integration constant that will be determined later. This equation indicates that the phase of the propagating waves takes on a nontrivial form, resulting in the emergence of nonlinear waves with a frequency chirp in the system.

The frequency chirp \( \Delta \omega = -i \theta_x / \delta x \) associated with the chirped wave solution can be expressed as

\[ \Delta \omega(x, t) = \frac{\sigma}{4(\alpha - \beta \nu)} U^2 - \frac{A}{2(\alpha - \beta \nu) U^2} + \frac{\beta \omega - v + \epsilon}{2(\alpha - \beta \nu)}. \tag{6} \]

From the relation (6), one can see that the frequency chirp includes two nonlinear contributions which are dependent on the wave intensity \( |q(x, t)|^2 = |U(\xi)|^2 \). The first nonlinear contribution that is directly proportional to the wave intensity is related to the nonlinear dispersion parameter \( \sigma \). For the case when the nonlinear dispersion effect is negligible (\( \sigma = 0 \)), the chirp-free nonlinear waves can be obtained if choosing the integration constant \( A = 0 \). Here we discuss the general case when the nonlinear dispersion effect has an influence on the dynamics of propagating waves (\( \sigma \neq 0 \)) and take \( A \neq 0 \).

Upon substituting Eq. (5) in Eq. (3), we obtain the following differential equation:

\[ U_{\xi \xi} + \frac{\mu}{(\alpha - \beta \nu)} U (U^2)_{\xi \xi} + \frac{3 \sigma^2 + 16 \alpha (\alpha - \beta \nu)}{16(\alpha - \beta \nu)^2} U^5 \]

\[ + \frac{2 \gamma (\alpha - \beta \nu) - \sigma (\nu - \beta \omega - \epsilon)}{2(\alpha - \beta \nu)^2} U^3 \]

\[ = 0. \]
which describes the dynamics of the wave amplitude in the weakly nonlinear nonlinear medium. By multiplying the amplitude equation (7) by the function $dU/d\xi$, one can integrate it once with respect to $\xi$ to obtain:

\[

\begin{align*}
\left(\frac{dU}{d\xi}\right)^2 + \frac{\mu}{2(\alpha - \beta v)} \left(\frac{dU}{d\xi}\right)^2 &= 4(\alpha - \beta v)^2 \\
36 + 2\gamma (\alpha - \beta v) - \frac{\mu}{2(\alpha - \beta v)} &= 4(\alpha - \beta v)^2 \\
\left(\frac{dU}{d\xi}\right)^2 + \frac{\mu}{2(\alpha - \beta v)} &= 4(\alpha - \beta v)^2 + 2\Gamma = 0,
\end{align*}
\]

where $\Gamma$ is the second integration constant.

We now make the transformation $\bar{F} = U^2$ and write Eq. (8) as

\[

\left(\frac{dF}{d\xi}\right)^2 + aF \left(\frac{dF}{d\xi}\right)^2 + bF^4 + cF^3 + dF^2 + 8\Gamma F + R = 0,
\]

where

\[

\begin{align*}
a &= \frac{2\mu}{\alpha - \beta v}, \\
b &= \frac{3\sigma^2 + 16\lambda(\alpha - \beta v)}{12(\alpha - \beta v)^2}, \\
c &= \frac{2\gamma (\alpha - \beta v) - \sigma (\alpha - \beta v)}{2(\alpha - \beta v)}, \\
d &= \frac{\sigma}{2(\alpha - \beta v)}, \\
R &= \frac{A^2}{(\alpha - \beta v)^2}.
\end{align*}
\]

Substituting the wave amplitude $U = F^{1/2}$ into Eq. (2), we find that the general traveling wave solution of the higher-order NLS equation (1) can be represented in the form:

\[

q(x, t) = \pm F^{1/2} (\xi) \exp \left[ i (\theta(\xi) + \omega t) \right],
\]

where $\theta(\xi)$ can be determined explicitly from Eq. (5) as,

\[

\begin{align*}
\theta(\xi) &= -\frac{\sigma}{4(\alpha - \beta v)} \int_{\xi_0}^{\xi} F(\xi) d\xi + \frac{A}{2(\alpha - \beta v)} \int_{\xi_0}^{\xi} \frac{1}{F(\xi)} d\xi \\
&+ \frac{\nu - \beta \omega - \epsilon}{2(\alpha - \beta v)} (\xi - \xi_0) + \theta_0,
\end{align*}
\]

with $F(\xi)$ obeying the nonlinear differential equation (9). Here, the arbitrary constant $\xi_0$ can be chosen freely and $\theta_0$ represents the initial phase.

The nonlinear differential equation (9) will now be solved to obtain exact propagating envelopes of the higher-order NLS equation (1). But closed form solutions of this equation cannot be obtained directly easily due to the existence of nonlinear terms with the range of power covering from 0 to 4 coexisting with another nonlinear term $F(\xi)^2$. In the following, we will use a suitable ansatz to obtain analytical periodic wave solutions that exhibit a nonlinear frequency chirp in the presence of all higher-order effects, as described by the model (1). In particular, the Jacobi elliptic function $dn$ is used to express the periodic nonlinear wave solutions presented here. A ‘gray’ localized solitary wave with frequency chirp is also determined in a long wave limit.

3. Results and discussion

To find the exact periodic wave solutions of Eq. (9), we will employ a suitable ansatz, which can be stated as follows,

\[

F(\xi) = S - P \frac{1}{dn^2(\eta(\xi - \xi_0), k)},
\]

where $S, P$ and $\eta$ are the wave parameters to be determined, and $dn(z, k)$ is Jacobi elliptic function of the third kind of modulus k taking values $0 < k < 1$. Note that the choice of this particular ansatz is dictated by the application of the homogeneous balance principle [see details in Ref. [38]] to Eq. (9) (namely, balancing the highest-order derivative term $F(\xi)^2$ with the higher power nonlinear term $F^4$ in Eq. (9)).

By plugging in the aforementioned ansatz into Eq. (9), and subsequently equating the coefficients of $dn^4(\eta(\xi - \xi_0), k)$, where $n$ takes on the values of $0, 2, 4, 6,$ and $8$, one can derive a set of algebraic equations in the following form:

\[

\begin{align*}
bS^4 + cS^3 + dS^2 + 8S + R &= 0, \\
bP^4 + cP^3 + dP^2 + 8P + R &= 0, \\
bP^2 + cP + d &= 0, \\
bP + c &= 0.
\end{align*}
\]

These algebraic equations can be solved to give the wave parameters $P, S$ and $\eta$ as,

\[

\begin{align*}
P &= -\frac{4\eta^2}{b}, \\
S &= \frac{1}{3b} \left[ \frac{b - \frac{ac}{a}}{a} - 4a(2 - k^2)\eta^2 \right], \\
\eta &= \frac{1}{2a} \left[ \frac{a^2c^2 + abc - 2b^2 - 3ba^2}{4k^2 - 4k + 1} \right]^{1/4},
\end{align*}
\]

with $F(\xi)$ obeying the nonlinear differential equation (9). Here, the arbitrary constant $\xi_0$ can be chosen freely and $\theta_0$ represents the initial phase.

The nonlinear differential equation (9) will now be solved to obtain exact propagating envelopes of the higher-order NLS equation (1). But closed form solutions of this equation cannot be obtained directly easily due to the existence of nonlinear terms with the range of power covering from 0 to 4 coexisting with another nonlinear term $F(\xi)^2$. In the following, we will use a suitable ansatz to obtain analytical periodic wave solutions that exhibit a nonlinear frequency chirp in the presence of all higher-order effects, as described by the model (1). In particular, the Jacobi elliptic function $dn$ is used to express the periodic nonlinear wave solutions presented here. A ‘gray’ localized solitary wave with frequency chirp is also determined in a long wave limit.

The corresponding chirping associated with this periodic wave can be obtained readily as

\[

q(x, t) = \pm \left[ S - P \frac{1}{dn^2(\eta(\xi - \xi_0), k)D^{1/2}} \right] \exp \left[ i (\theta(\xi) + \omega t) \right].
\]
\[ \Delta \omega (x, t) = \frac{\sigma}{4(\alpha - \beta v)} \left[ S - P \operatorname{dn}^2 \left( \eta (\xi - \xi_0) \right) \right] \]
\[ \quad - \frac{A}{2(\alpha - \beta v)} \left[ S - P \operatorname{dn}^2 \left( \eta (\xi - \xi_0) \right) \right] \]
\[ \quad + \frac{\beta \omega - v + \epsilon}{2(\alpha - \beta v)}, \]
\quad \text{(25)}

Fig. 1(a) depicts the intensity profile of the chirped \( \operatorname{dn}^2 \) periodic wave solution (24) for the parameters values: \( \alpha = 0.8, \beta = -2, \gamma = -0.5, \lambda = -2.25, \sigma = 2, \epsilon = 0.1, \omega = 0.5, \mu = 0.125, \delta = 5.25, \nu = 0.1, \) and \( k = 0.25. \)

Considering the long-wave limit \( k \to 1, \) the chirped periodic wave solution (24) degenerates to a chirped solitary wave solution of the form:

\[ q(x, t) = \pm \left[ S_0 - P_0 \operatorname{sech}^2 \left( \eta_0 (\xi - \xi_0) \right) \right]^{1/2} \exp \left[ i \left( \theta(\xi) + \omega t \right) \right]. \]
\quad \text{(26)}

with \( S_0 > P_0 > 0 \) and \( \eta_0 > 0. \)

In this solution, the solitary wave parameters \( P_0, S_0 \) and \( \eta_0 \) are given by:

\[ P_0 = -\frac{4a\eta_0^2}{b}, \]
\[ (27) \]
\[ S_0 = \frac{1}{3b} \left( \frac{b - ac}{a} - 4a\eta_0^2 \right), \]
\[ \eta_0 = \frac{1}{2a} \left[ \frac{a^2c^2 + abc - 2b^2 - 3bda^2}{a} \right]^{1/4}. \]
\quad \text{(28)}

Furthermore, the integration constants \( \Gamma \) and \( R \) are

\[ \Gamma = \frac{5}{8} \left[ \frac{5}{3} \left( \frac{16a\eta_0^2}{a} - 4b + 5ac \right) - 2d \right], \]
\quad \text{(30)}
\[ R = \frac{S_0^2}{S_0} \left[ \frac{b + ac}{a} - 4a\eta_0^2 \right] + d. \]
\quad \text{(31)}

Therefore, the corresponding solitary wave intensity reads

\[ |q(x, t)|^2 = S_0 - P_0 \operatorname{sech}^2 \left[ \eta_0 (x - vt - \xi_0) \right]. \]
\quad \text{(32)}

Here, we assumed that the constraint conditions \( a > 0 \) and \( a^2c^2 + 3b^2 + 3bda > 0 \) are satisfied for the parameter \( \eta_0 \) in (29) to be real and positive. Moreover, the requirement \( b < 0 \) is also necessary for the parameter \( P_0 > 0 \) in (27). In this situation, the positivity of \( S_0 \) also requires \( b - ac < 0 \) in Eq. (28).

Upon analyzing the chirped solution (26), we can observe that its amplitude may approach nonzero value when the distance variable \( (\xi) \) tends to infinity. Additionally, it is apparent that this particular class of chirped localized solution has no free parameters and that its characteristics, including background, width, and amplitude, are solely determined by the system parameters.

The evolution of the intensity profile for the chirped soliton (26) is depicted in Fig. 2(a) for \( \beta = -2, \alpha = 0.8, \gamma = -2.0, \epsilon = 0.1, \sigma = 2, \omega = 0.5, \mu = 0.125, \delta = 5.25, \nu = 0.1, \) and \( \xi_0 = 0. \) We can observe from this figure that the localized structure appears as a gray solitary wave on a continuous-wave background.

The frequency chirp associated with this gray solitary pulse can be obtained by:

\[ \Delta \omega (x, t) = \frac{\sigma}{4(\alpha - \beta v)} \left[ S_0 - P_0 \operatorname{sech}^2 \left( \eta_0 (\xi - \xi_0) \right) \right] \]
\[ \quad - \frac{A}{2(\alpha - \beta v)} \left[ S_0 - P_0 \operatorname{sech}^2 \left( \eta_0 (\xi - \xi_0) \right) \right] \]
\[ \quad + \frac{\beta \omega - v + \epsilon}{2(\alpha - \beta v)}. \]
\quad \text{(33)}

Equation (33) demonstrates that the nonlinear dispersion coefficient \( \sigma \) affects the two intensity dependent chirping terms explicitly as well as implicitly through its appearance in the constant \( A. \) When \( A \) is zero, the frequency chirp \( \Delta \omega \) will vary as directly proportional to the intensity of the wave and its magnitude can be adjusted by varying the coefficient \( \sigma. \) One notes that in the absence of nonlinear dispersion, the frequency chirp will be reduced to a linear function of \( x \) if \( A \) is set to zero. Therefore, we can infer that the nonlinearity in pulse chirp here is essentially connected to the nonlinear dispersion effect. It is intriguing to investigate the impact of this process on the frequency chirp of propagating localized waves numerically. Fig. 2(b) illustrates the profile of frequency chirp \( \Delta \omega \) across the pulse of gray solitary wave at \( t = 0 \) for different values of coefficient \( \sigma: 1.6, 1.8, 2. \) Here we have taken the same parameter values as those in Fig. 2(a). We observe that the magnitude of frequency chirp increases continuously with the decreasing nonlinear dispersion coefficient \( \sigma. \) It is relevant to see that this parameter also influences the background intensity of the chirped gray solitary wave. Therefore, we may conclude that the nonlinear dispersion parameter \( \sigma \) enables efficient control of the
magnitude of the chirp as the pulse propagates through the optical waveguide.

4. Stability analysis

Having obtained the nonlinearly chirped gray solitary waves of the underlying equation (1), it is crucial to investigate their stability with respect to finite perturbations. This is because only stable or weakly unstable solitons can be observed experimentally and utilized in practical applications [39]. It is therefore necessary to study the stability of these nonlinearly chirped waves against finite initial perturbations, which may be slight violation of the parametric conditions, amplitude perturbation, and random noises [40]. It should be noted that important results have been obtained with previous theoretical studies concerning stability properties of solitary pulses in systems with cubic nonlinearity [41]. Moreover, it was found that competing nonlinearities appearing in nonlinear cubic-quintic media can stabilize soliton solutions [42]. The envelope equation (1) includes such cubic-quintic nonlinearities, in addition to nonlinear dispersion and weak nonlocality. It is therefore not unreasonable to conjecture that the solitary pulse solutions presented here are stable. However, a detailed analysis is required in order to strictly answer the issue of stability of such privileged chirped solutions. In the following, we analyze the stability of the chirped solutions with respect to the finite perturbations by employing numerical simulations.

Here, we performed two types of direct numerical simulations with amplitude perturbation and initial white noise [43,44], to study the stability of the chirped gray solitary wave solution (24) compared to Fig. 2(a). First, we perturb the amplitude (10%) in the initial distribution. The numerical results are shown in Fig. 3(a) in which the amplitude in the initial distribution is perturbed. Here the material parameters adopted are the same as in Fig. 2(a). From this figure, one can clearly see that the perturbation of amplitude could not influence the main character of the solution. Second, we add white noise (10%) in the initial pulse. The evolution plot of chirped solitary-wave solutions (24) under the perturbation of 10% white noise is depicted in Fig. 3(b). The results show that the chirped gray solitary wave can propagate in a stable fashion in the nonlinear medium under finite initial perturbations of the additive white noise. Therefore, our chirped solitary waves show structural stability with respect to the small input profile perturbations. Thus we can conclude that the nonlinearly chirped gray solitary waves we obtained are stable.

5. Conclusions

To summarize our findings, we have examined the existence of chirped periodic and solitary waves in a weakly nonlinear material with parabolic law nonlinearity and spatio-temporal dispersion. We have found that a family of $dn^2$-type periodic waves can be formed in the system in the presence of all physical processes. This class of optical nonlinear waveforms was shown to exhibit a nonlinear chirp, which includes two intensity dependent nonlinear contributions. We have also shown that this frequency chirping property results from the nonlinear dispersion effect. In addition, we have presented the long-wave limit of the obtained periodic wave solution, which has given rise to a class of gray-type solitary wave solutions. The stability of the chirped solitary waves has been also demonstrated numerically with respect to finite perturbations of the additive white noise and perturbation of the amplitude. The results showed that the solutions we obtained are still stable under finite initial perturbations, such as amplitude and white noise. Due to their robust nature, the obtained chirped gray solitary waves should be observed experimentally in nonlocal nonlinear materials exhibiting a rich variety of physical, including spatio-temporal dispersion, parabolic law nonlinearity, detuning, and nonlinear dispersion.

To describe more realistic phenomena, the inhomogeneities present in the nonlinear medium have to be considered. In this situation, we have to extend the results obtained here within the framework of the constant-coefficient higher-order NLS equation (1) and consider the generalized higher-order NLS equation with time-dependent coefficients. The study of evolutionary dynamics of chirped soliton pulses in such nonautonomous system is the subject of future investigation.

CRediT authorship contribution statement


Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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